Inverse Inference on Cooperative Control of Networked Dynamical Systems

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Abstract

Recent years have witnessed the rapid advancement of understanding the control mechanism of networked dynamical systems (NDSs), which are governed by components such as nodal dynamics and topology. This paper reveals that the critical components in continuous-time state feedback cooperative control of NDSs can be inferred merely from discrete observations. In particular, we advocate a bi-level inference framework to estimate the global closed-loop system and extract the components, respectively. The novelty lies in bridging the gap from discrete observations to the continuous-time model and effectively decoupling the concerned components. Specifically, in the first level, we design a causality-based estimator for the discrete-time closed-loop system matrix, which can achieve asymptotically unbiased performance when the NDS is stable. In the second level, we introduce a matrix logarithm based method to recover the continuous-time counterpart matrix, providing new sampling period guarantees and establishing the recovery error bound. By utilizing graph properties of the NDS, we develop least square based procedures to decouple the concerned components with up to a scalar ambiguity. Numerical simulations demonstrate the effectiveness of the proposed method.

I. Introduction

In the last decades, networked dynamical systems (NDSs) have played a crucial role in many engineering and biological fields, e.g., multi-robot formation, power grids, human brain, and immune cell network. An NDS, comprising multiple interconnected nodes, is characterized by not only the self-dynamics of a single node (nodal dynamics) but also the interaction structure (topology) between nodes, and can achieve various cooperative behaviors such as synchronization. However, the prior information about the nodal dynamics and topology is not always accessible in practice, and needs to be inferred from observations. This inference enhances our ability to understand, predict, and intervene with the NDS. This paper focuses on the continuous-time linear state-feedback cooperative control of NDSs, where only discrete and noisy observations on a single round of the system's trajectory are available. In particular, we aim to provide a systematic approach for inferring both the nodal dynamics and the topology of the NDS, and subsequently reconstructing the control objective function that governs the process.

II. Main Results

First, consider N nodes in a NDS $\mathcal{G}=(\mathcal{V},\mathcal{E})$ share a common state space $\mathbb{R}^n$ and aim to cooperatively accomplish a task. Let $x_i \in \mathbb{R}^n$ be the state of node i, subject to the following identical linear time-invariant (LTI) dynamics

$$\begin{cases} \dot{x}_i = Ax_i + Bu_i \\ u_i = K\sum_{j\in\mathcal{N}_i} a_{ij}\left(x_j - x_i\right) \end{cases}, \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are the nodal system matrices, $u_i \in \mathbb{R}^m$ is the control input of node i, a_{ij} is the weight of the directed edge from node j to i, and $K \in \mathbb{R}^{m\times n}$ is the feedback gain matrix.

For the external observer, it can observe the state trajectory of the NDS with a sampling period $\tau > 0$ and zero-order holder. Denote $x_i(k)$ as node i's state at instant $k\tau$, and the corresponding observation is given by

$$y_i(k) = x_i(k) + \upsilon_i(k), \tag{2}$$

where $\upsilon_i(k) \sim \mathbf{N}(0,\Gamma)$ is an independent Gaussian noise vector, with $\Gamma = \mathrm{diag}\{\sigma_{\upsilon,1}^2, \cdots, \sigma_{\upsilon,n}^2\}$ being the covariance matrix for n different state dimensions. The corresponding discrete-time model that the observer relies on is given by

$$\begin{cases} x(k+1) = e^{A_c\tau}x(k) \triangleq A_d x(k) \\ \quad y(k) = x(k) + \upsilon(k) \end{cases}, \tag{3}$$

where $y=[y_1^\intercal, y_2^\intercal, \cdots, y_N^\intercal]^\intercal \in \mathbb{R}^{Nn}$, $\upsilon=[\upsilon_1^\intercal, \upsilon_2^\intercal, \cdots, \upsilon_N^\intercal]^\intercal \in \mathbb{R}^{Nn}$, and the matrix exponential A_d represents the discretized closed-loop system matrix.

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Based on the above formulation, our key idea is to first construct the closed-loop system matrix A_d of the NDS from noisy observations $\{y(k)\}_{k=1}^{T}$, and then extract the relevant components $\{A, B, L, K\}$ by exploiting the cooperative nature. The contributions are summarized as follows.

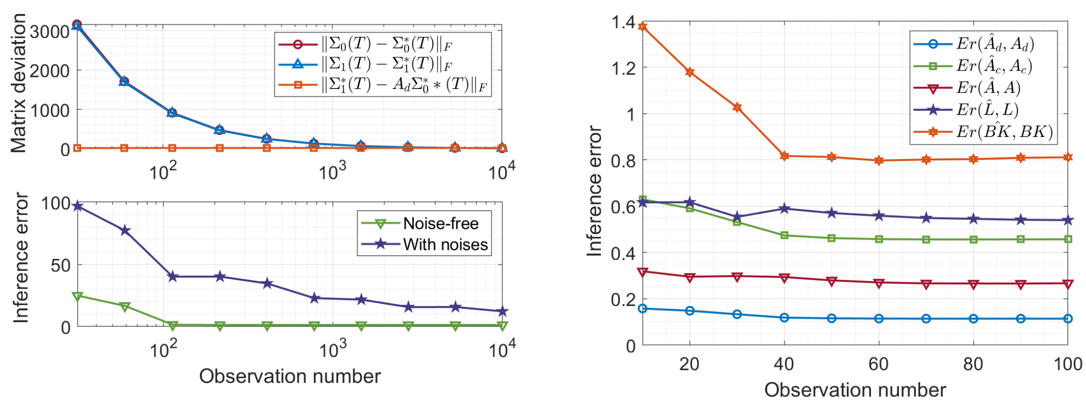
- We derive conditions of inferring the control mechanism of NDSs through observation driven methods. Precisely, we propose a bi-level inference framework to infer the nodal dynamics and the topology in continuous-time state-feedback cooperative control, along with theoretical guarantees.
- At the first level, we design a causality based estimator for the discretized closed-loop system matrix. This estimator effectively bridges the gap between discrete observations and the continuous-time model, achieving unbiased accuracy in the asymptotic sense when the NDS is stable.
- We develop matrix logarithm and least squares based procedures to decouple the concerned components in the second level. Sufficient conditions regarding the sampling period for continuous-time model estimation are derived, along with the corresponding error bound. Notably, the topology and feedback gain can be accurately inferred up to a scalar ambiguity.

III. Simulation Results

We consider a NDS of 6 nodes subject to the model (1), where the adjacency topology $\mathbf{A}_0$, nodal matrices A and B, and the feedback gain K are given by

$$\mathbf{A}_0 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \; A = \begin{bmatrix} -1 & 2 & 5 \\ 1 & -1 & 2 \\ -5 & 0 & -1 \end{bmatrix}, B^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \; K = \begin{bmatrix} -0.0365 & 0.4295 & 0.9216 \end{bmatrix}.$$

The initial states of each node in each dimension are randomly generated in $[0, 1000]$. The state trajectory starts from 0-50s.



(a) Asymptotic inference performance on A_d.

(b) Inference errors of A_d, A_c, A and BK.

Fig. 1. Performance under $\tau = 0.05$s and noise setting G_2.

First, since the inference for A_d is fundamental for the entire method, we present the asymptotic performance of the naive estimator $\widehat{A}_d(T) = \Sigma_1(T)\left(\Sigma_0(T) - I_N \otimes \Gamma\right)^{-1}$ in Fig. 1(a). In this test, the sampling period is set to $\tau = 0.05$s. For clarity, the top subplot shows the deviation between the noisy sample matrix $\Sigma_0(T)$ and $\Sigma_1(T)$) versus their noise-free counterparts $\Sigma_0^*(T) = (X^-(X^-)^\intercal)/T$ and $\Sigma_1^*(T) = (X^+(X^-)^\intercal)/T$, along with the residual $\|\Sigma_1^*(T) - A_d\Sigma_0^*(T)\|_F$. The plotted curves clearly indicate that both $\Sigma_0(T)$ and $\Sigma_1(T)$ will approach their noise-free counterparts asymptotically. Similarly, in the bottom subplot, the performance of the naive estimator also converges the noise-free version asymptotically. The naive estimator is considered an ideal design primarily in an asymptotic sense.

Next, we provide the inference errors of all concerned parameters regarding the observation number with $\tau = 0.05$. Fig. 1(b) presents the inference results of A_d, A_c, A and BK in a noisy setting. In these plots, the inference errors of the concerned matrices will generally decrease with T initially, but remain slightly changed as T becomes larger. Although this pattern may not correspond to our typical expectation that the errors should approach zero asymptotically, it is important to note that this behavior is largely attributable to numerical precision limitations inherent in computational processes (similar trends are noted even in noise-free scenarios). As the system state approaches zero in a short time, subsequent observations are almost zero-mean noises, contributing little to the accuracy of the estimator.