Water Distribution Leakage Localization and One-Way Pipes

Victor Molnö, Henrik Sandberg, KTH Royal Institute of Technology, vmolno@kth.se

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Water distribution is a fundamental societal infrastructure function. To *Ensure access to clean water and sanitation for all* is the United Nations' sixth sustainable development goal [1]. Unfortunately, water distribution systems suffer heavily from leakages. According to statistics, globally roughly 30% of the input water volume is lost due to leaks [2]. Consequently, leakage localization is an important task in water distribution system management.

Historically, water leakage localization has required extensive manual labor, and so with a recently increasing number of smart, integrated and connected sensors (measuring pressure and flow), there has been a surge in efforts to automate the leakage localization process. There are some examples of sensor-based leakage localization schemes implemented in practice (see [3]). However, there are even more academic examples that are not necessarily limited to the current sensor configuration. These examples also explore potential possibilities—imagining what could be achieved if additional sensors were installed. An influential publication in the field is the *Battle of the Leakage Detection and Isolation Methods* (BattLeDIM) competition [4], which includes many innovative approaches. Another wide survey can be found in [5], recommended to anyone who is getting into the subject.

Now while, there are many clever preactical approaches, the fundamental theory of leakage localization is still incomplete. Such a theory should explain, for instance, which sensors are needed to localize a leakage, whether certain hydraulic states make localization easier, and the risk of finding the wrong leak position.

In this work, we give a sufficient condition for when leaks can be reliably located. The work was presented at CDC 2024 [6], and also part of the licentiate thesis [7].

Remark. While the existing leakage localization theory is incomplete we do not claim that there are no previous works of this kind. For an early contribution we suggest to look at [8].

Model Description and Leakage Hypothesis Testing Procedure

The standard model underlying all related works is a steady-state, graph-based potential flow model.

Remark. there exists leakage localization research with dynamical models [9], however, this is not our focus.

The steady state model consists of two equations

$$A\mathbf{q} = \mathbf{d}, \tag{1}$$

$$A^T\mathbf{h} = -\mathbf{U}(\mathbf{q}). \tag{2}$$

Here, the *demands* $\mathbf{d} = (d_1, \ldots, d_n) \in \mathbb{R}^n$, the hydraulic heads $\mathbf{h} \in (h_1, \ldots h_n) \in \mathbb{R}^n$, and the *flows* $\mathbf{q} = (q_1, \ldots, q_m) \in \mathbb{R}^m$ where n is the number of nodes $V = \{v_i\}_{i=1}^n$ and m is the number of pipes $E = \{e_j\}_{j=1}^m$ in the network. The incidence matrix A has the elements

$$A_{ij} = \begin{cases} -1 & \text{if pipe } e_j \text{ starts in node } v_i, \\ 1 & \text{if pipe } e_j \text{ ends in node } v_i, \\ 0 & \text{otherwise.} \end{cases}$$

The mass-balance equation (1) says that the demand in a node is the difference between incoming and outgoing flow. The transport equation 2 describes how the hydraulic head drops over the pipes at certain flows. The so-called *head-loss functions* $\mathbf{U}(\mathbf{q}) = (\mathbf{U}_1(q_1), \ldots, \mathbf{U}(q_m))$ are increasing functions of pipe flow.

In case there is no leakage, i.e., $\sum_{i=1}^{n} d_i = 0$, there is theory [10] that guarantees a unique solution (up to an additive hydraulic head constant) to (1)-(2) given $\mathbf{d}$. Unfortunately, as discussed above, there is always a leakage. In this work, we model a leak as point $\Lambda^* = (e_{k^*}, x)$ in a pipe e_{k^*}, where $x \in [0,1]$ is the relative position of the leak along e_{k^*}. The task for the operator is to localize the leakage, i.e., to determine Λ^*. To their help, we assume access to all demands $\mathbf{d}$ and the hydraulic heads $\mathbf{h}_S$ in some subset of nodes, $S \subseteq V$.

The standard approach to leakage localization (somehow used by most entries in BattLeDIM [4] among others), is to pose a leak location hypothesis $\Lambda = (e_j, x_j)$. Under this hypothesis, a fictional node is added in Λ. The incidence matrix A and head-loss functions $\mathbf{U}$ are appropriately updated. The leak demand $d_{n+1}^{\Lambda} = d_{\text{leak}} = -\sum_{i=1}^{n}$ is added to the new node in Λ. This gives a new set of equations (1)-(2), where indeed $\sum_{i=1}^{n+1} d_i^{\Lambda} = 0$. As we assumed access to $\mathbf{d}$, the operator then uses the theory [10] and numerical methods to solve for $\mathbf{h}^{\Lambda}$, i.e., the hydraulic heads under leakage position hypothesis Λ. To evaluate Λ, the calculated $\mathbf{h}_S^{\Lambda}$ is compared to the measured $\mathbf{h}_S$. If these differ, the hypothesis may be rejected. If they are identical, we have reason to believe $\Lambda = \Lambda^*$, however can we be sure?

Although other works, as well as we do, notice cases where two different hypotheses Λ_1 and Λ_2 yield the same $\mathbf{h}_S^{\Lambda}$, the phenomena has not been thoroughly investigated. In this work we derive conditions for when we can be sure to avoid this pitfall, i.e., that leakage localization breaks down due to false but plausible leakage location hypothesis.

One-Way Pipes and Main Result

We introduce something we call *one-way* pipes.

Definition 1. We say that pipe e_j is *one-way* between nodes v_i and v_l if both

1. there is a path on the graph from v_i to v_l through e_j,
2. there is *no* path on the graph from v_i to v_l backwards through e_j.

Our main result can be summarized as in Theorem 1.

Theorem 1. *Assume $v_i, v_l \in S$. If one or more one-way pipes between v_i and v_l follow in a path π, there will be at most one plausible ($\mathbf{h}_S^{\Lambda} = \mathbf{h}_S$) leakage position hypothesis Λ in π. Consequently, if $\Lambda^* \in \pi$, we will find it with measurements h_i, h_j.*

Theorem 1 is a first analytical step (sufficient condition) to relate the expected success of leakage localization to the pressure sensor distribution S. However, there is lot more work to do to understand exactly when we get false but plausible leakage positions.

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