Low-Complexity Control for Handling Generalized Time-Varying Output Constraints in Uncertain MIMO Nonlinear Systems

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I. Introduction

This work investigates closed-form feedback control designs under time-varying output constraints, a critical aspect of nonlinear control systems essential for ensuring tracking and stabilization performance as well as safety requirements. Existing closed-form feedback methods addressing time-varying constraints fall into three primary categories: Funnel Control (FC) [1], Prescribed Performance Control (PPC) [2], and Time-Varying Barrier Lyapunov Function (TVBLF) methods [3]. Typically, these control methods are used to attain user-defined transient and steady-state performance for tracking and stabilization errors in nonlinear dynamical systems. Despite various successful applications, these methods have limitations in managing couplings among multiple time-varying constraints. FC, PPC, and TVBLF can only enforce funnel-type constraints (e.g., $-\rho_i(t) < e_i < \rho_i(t)$) on independent states or errors, inherently resulting in decoupled constraints representing box-shaped sets in output or error spaces. However, practical scenarios frequently require addressing arbitrary and potentially coupled time-varying output constraints, particularly for safety or spatiotemporal specifications [4].

In response, we propose a novel feedback control strategy designed to satisfy potentially coupled time-varying output constraints for uncertain, high-relative degree MIMO nonlinear systems. Inspired by [4], our method integrates all time-varying constraints into a single consolidated constraint. To ensure compliance with this unified constraint, we introduce a robust yet low-complexity control approach motivated by [5]. Importantly, this method avoids relying on approximations or parameter estimation schemes for handling uncertainties. Additionally, by adaptively updating the consolidated constraint online, our solution achieves a least violating solution if the constraints become infeasible over unknown time intervals.

Unlike FC, PPC, and TVBLF, our method effectively handles both generic asymmetric funnel-type and one-sided time-varying constraints, thus addressing broader spatiotemporal specifications. Moreover, while conventional methods require initial satisfaction of all output constraints, our approach guarantees convergence to and the invariance the output-constrained set within a user-defined finite time, even from initially infeasible conditions. This extended abstract summarizes recent results reported in [6].

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II. Problem Formulation

Consider high-relative degree uncertain nonlinear MIMO systems described as:

$$\begin{aligned} \dot{x}_i &= f_i(t,\bar{x}_i) + G_i(t,\bar{x}_i)x_{i+1}, \quad i \in \{1,\dots,r-1\},\\ \dot{x}_r &= f_r(t,\bar{x}_r) + G_r(t,\bar{x}_r)u, \\ y &= h(t,x_1), \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^n$, $\bar{x}_i := [x_1^\top,\dots,x_i^\top]^\top \in \mathbb{R}^{ni}$, $u \in \mathbb{R}^n$, and $y \in \mathbb{R}^m$. Functions f_i and matrices G_i are locally Lipschitz and unknown and h is a $\mathcal{C}^2$ map in x_1 and $\mathcal{C}^1$ in t. Moreover, for each fixed $\bar{x}_i$, entries of f_i and G_i are bounded for all $t \geq 0$. Similarly, elements of h are bounded for all $t \geq 0$ when x_1 is fixed. The output y must satisfy generalized time-varying constraints:

$$\underline{\rho}_j(t) < h_j(t,x_1) < \overline{\rho}_j(t), \quad \forall t \geq 0, \tag{2}$$

where $j \in \{1,\dots,m\}$, and $\underline{\rho}_j(t), \overline{\rho}_j(t)$ are bounded and continuously differentiable. Note that (2) describes funnel constraints (when both $\underline{\rho}_j(t)$ and $\overline{\rho}_j(t)$ exist), upper one-sided constraints (if only $\overline{\rho}_j(t)$ exists), and lower one-sided constraints (if only $\underline{\rho}_j(t)$ exists).

Note that the output constraints specified in (2) depend on x_1, which signifies the spatial coordinates (positions) of mechanical systems. Moreover, the output map $h(t,x_1)$ is employed only to model various types of spatiotemporal constraints for the nonlinear dynamics (1). In our work, $h(t,x_1)$ is not necessarily related to the available measurements of the system and we assume all states of (1) can be measured.

Let $\bar{\Omega}(t)$ represents the time-varying output constrained set that include all constraints in (2):

$$\bar{\Omega}(t) := \{x_1 \in \mathbb{R}^n \mid \underline{\rho}_j(t) < h_j(t,x_1) < \overline{\rho}_j(t)\}. \tag{3}$$

Our goal is to design a low-complexity continuous robust feedback control law $u(t,x)$ for (1) such that $x_1(t;x(0),u)$ satisfies the time-varying output constraints (2) $\forall t > T \geq 0$, where T is a user-defined finite time after which the output constraints are satisfied for all time (i.e., $x_1(t;x(0)) \in \bar{\Omega}(t), \forall t > T \geq 0$). Note that this problem reduces to establishing only invariance of $\bar{\Omega}(t)$ for all $t \geq 0$, if $x_1(0) \in \bar{\Omega}(0)$ ($T=0$). On the other hand, having $x_1(0) \notin \bar{\Omega}(0)$ indicates establishing: (i) finite time convergence to $\bar{\Omega}(t)$ at $t=T$, and (ii) ensuring invariance of $\bar{\Omega}(t)$, for all $t > T$.

III. Controller Design Methodology

To encode all different types of time-varying constraints in (2) into a single constraint, first, we rewrite all constraints in (2) in the form $\psi_j(t,x_1) > 0$ where

$$\psi_j(t,x_1) = h_j(t,x_1) - \underline{\rho}_j(t) \quad \text{or} \quad \overline{\rho}_j(t) - h_j(t,x_1). \tag{4}$$

Using the Log-Sum-Exp function, we define

$$\alpha(t,x_1) = -\frac{1}{\nu}\ln\left(\sum_{i=1}^{m+p} e^{-\nu\psi_i(t,x_1)}\right) \leq \min\{\psi_1(t,x_1),\ldots,\psi_{m+p}(t,x_1)\}, \tag{5}$$

where p denotes the funnel-type constraints in (2). Clearly, $\alpha(t,x_1) > 0$ at time t ensures all time-varying constraints are satisfied at t. Thus, the consolidated constraint for (1) is defined as:

$$\alpha(t,x_1(t)) > \rho_\alpha(t), \quad \forall t \geq 0, \tag{6}$$

where $\rho_\alpha(t)$ is suitably designed (see [6] for details) to guarantee finite-time convergence to and/or invariance of the time-varying constrained set $\bar{\Omega}(t)$ for $t \geq T$. Specifically, if $\alpha(0,x_1(0)) > 0$ (constraints initially satisfied) and all constraints in (2) remain feasible (i.e., $\bar{\Omega}(t)$ nonempty) at all times, one may choose $\rho_\alpha(t) = 0$. For potentially infeasible constraints (i.e., $\bar{\Omega}(t)$ in (3) may temporarily become empty), online tuning of $\rho_\alpha(t)$ is crucial. In such cases, we propose an estimation scheme for $\alpha^*(t) := \max_{x_1\in\mathbb{R}^n}\alpha(t,x_1) \leq \bar{\alpha}^*(t)$, where a positive value of $\alpha^*(t)$ indicates the feasibility of $\bar{\Omega}(t)$. Fig. 1 demonstrates the satisfaction of (6) under possible constraint infeasibility in (2). In Fig. 1, $\varrho(t)$ serves as a nominal lower bound for (6), while $\hat{\alpha}$ denotes the estimate of $\alpha^*(t)$. The lower bound $\rho_\alpha(t)$ exhibits adaptive behavior: it follows the desired nominal profile until the estimation signal conflicts with $\varrho(t)$. Thereafter, enforcing (6) along closed-loop trajectories yields a least-violating solution.

To establish the satisfaction of (6) along closed-loop trajectories of (1), a low-complexity (i.e., approximation-free) backstepping-like closed-form state feedback is designed as follows. First, define the error $e_\alpha(t,x_1) = \alpha(t,x_1) - \rho_\alpha(t)$ and apply a nonlinear transformation:

$$\epsilon_\alpha(t,x_1) = \ln\left(\frac{e_\alpha}{\upsilon}\right). \tag{7}$$

where υ is a tunable positive constant. To ensure constraints satisfaction, a time-varying barrier function-based virtual control is introduced:

$$s_1(t,x_1) = -k_1\frac{\epsilon_\alpha}{e_\alpha}\nabla_{x_1}\alpha(t,x_1). \tag{8}$$

where $k_1 > 0$ is a control gain. Then inspired by [5] we define recursively: $e_i(t,\bar{x}_i) = x_i - s_{i-1}(t,\bar{x}_{i-1}), \quad i = 2,\ldots,r$, and target enforcing narrowing intermediate funnel constraints $-\vartheta_{i,j}(t) < e_{i,j}(t,\bar{x}_i) < \vartheta_{i,j}(t), i \in \{2,\ldots,r\}, j \in \{1,\ldots,n\}$, for all $t \geq 0$, where $\vartheta_{i,j}(t) := (\vartheta_{i,j}^0 - \vartheta_{i,j}^\infty)\exp(-l_{i,j}t) + \vartheta_{i,j}^\infty$ to practically compensate the errors $e_{i,j}$. The following intermediate control laws ensure practical error compensation:

$$s_i(t,e_i) = -k_i\Xi_i\epsilon_i, \tag{9}$$

where ϵ_i transforms the normalized errors:

$$\epsilon_{i,j} = \ln\left(\frac{1+\hat{e}_{i,j}}{1-\hat{e}_{i,j}}\right), \quad \hat{e}_{i,j} = \frac{e_{i,j}}{\vartheta_{i,j}(t)}. \tag{10}$$

and $\Xi_i := \mathrm{diag}(\xi_{i,j}) := \frac{\partial\varepsilon_i(\hat{e}_i)}{\partial\hat{e}_i}\frac{\partial\hat{e}_i(t,e_i)}{\partial e_i} \in \mathbb{R}^{n\times n}$ is a diagonal matrix. Finally, the robust low-complexity feedback controller is designed as: $u(t,x) = s_r(t,x)$.

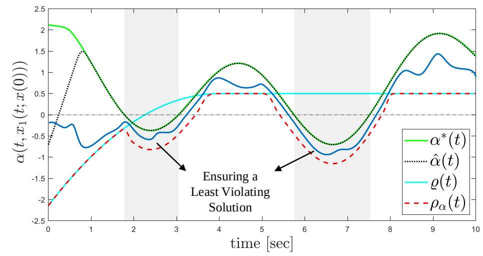

Fig. 1: The evolution of $\alpha(t,x_1(t;x(0)))$ under the consolidating constraint (6), under temporary infeasibility of the time-varying constraints.

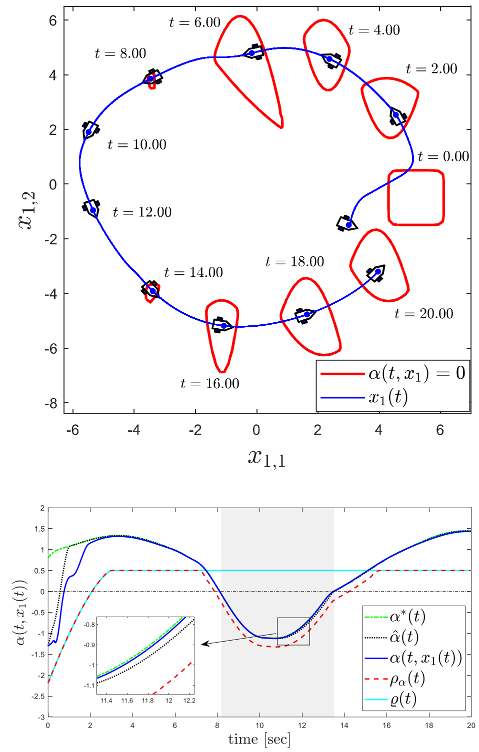

Fig. 2: Time-varying region tracking of a mobile robot.

To demonstrate the effectiveness of the proposed approach, we consider a region tracking problem for a mobile robot subject to both feasible and temporarily infeasible constraints, while ensuring robustness to dynamic uncertainties and external disturbances. Fig. 2 illustrates the simulation results.

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