# Analysis of Control Systems under Sensor Timing Misalignments

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Abstract-This paper presents an in-depth analysis of the stability and performance of control systems experiencing sensor timing misalignments, a common challenge in practical applications such as autonomous vehicles and aerospace systems. We model multichannel sensor delays as independent random variables, capturing the variability of real-world systems where different sensors exhibit distinct and non-constant processing times or communication delays. This allows us to formulate the systems subject to delays as Markov Jump Linear Systems, enabling a rigorous examination of stability and performance under such misalignments. To validate the proposed methodology, we conduct experimental studies with two applications: an adaptive cruise controller and an inverted pendulum. In the analysis, we show that the impact of delays in different channels on a control system is typically not symmetric. Very often there is a sensor channel that is more critical for the controller and delays in that specific channel are hardly tolerated, while other channels can be delayed without compromising the stability and performance of the system.

## I. INTRODUCTION

Cyber-physical systems use controllers to manage physical processes through discrete-time signals, crucial for maintaining stability and optimising performance, especially under disturbances or uncertainties. However, sensor data is not always perfectly synchronised, and timing misalignments can disrupt feedback loops, impairing stability and causing unexpected behaviour.

Recent research on autonomous vehicles [3] shows that misalignments across different sensors affect the accuracy of object detection pipelines, with fusion algorithms being moderately robust to image misalignments but highly sensitive to LiDAR misalignments. However, the study does not address the impact on control algorithms that use the misaligned data, which is the subject of this research.

We address this critical gap in existing research by rigorously analysing the effects of sensor timing misalignments on the stability and performance of control systems. We focus on scenarios where different sensors introduce independent, variable delays, a situation commonly found in real-world applications. Such delays can arise due to distinct processing times for different sensors, communication latencies, or computational requirements for data extraction.

Our paper titled Analysis of Control Systems under Sensor Timing Misalignments [4], which covers this topic in depth, will be presented at the 31st IEEE Real-Time and Embedded Technology and Applications Symposium (RTAS).



Fig. 1. The spectral radius  $\rho$  for the Furuta pendulum case study with combinations of constant delays  $d_1$  on the pendulum angle,  $d_2$  on the pendulum angular velocity, and  $d_3$  on the base angular velocity sensors. The delay that is not shown is kept constant at zero. Blue indicates stability, red indicates instability.

### II. METHODS

In this work, we model multi-channel delays as random variables, capturing the variability inherent in practical systems. We consider two distinct delay notions. On one hand, the *sensor delay* that each sample of the process output experiences before it can be used for control. We assume that the sensor delay is independent and identically distributed in each channel, and independent between channels. On the other hand, the *controller delay* is the delay experienced by the controller when sensing the output channels. It represents the age of the data that is currently used. Even though the sensor delay is the one that is typically characterised formally (e.g., using scheduling theory), the controller delay is the most useful for our analysis.

We show that the controller delay in each of the sensor channels is governed by a discrete-time Markov chain. By then formulating the closed-loop system as a Markov Jump Linear System (MJLS) [2], we enable a robust analysis framework that considers the stochastic nature of the delays. This approach allows us to derive mean square stability conditions based on the probability distributions of the sensor delays, providing theoretical guarantees for system stability in a range of delay scenarios. Additionally, we explore the performance implications of timing misalignments, examining how they affect the expected control cost in comparison to an idealised scenario with perfectly synchronised data.

# **III. RESULTS**

We introduce two case studies to demonstrate our analysis and validate our model: an adaptive cruise controller and a Furuta pendulum. Here, we only summarise the results related to the pendulum. The other case study is covered in [4].



Fig. 2. The normalised cumulative cost of the Furuta pendulum for three time-varying delay configurations. The sensor delays in each sensor channel are normally distributed with mean  $d_i$  and standard deviation 1, and truncated to avoid negative values.

The control of an inverted pendulum with rotating base, commonly known as the Furuta pendulum, is a well-known problem [1]. We focus on balancing the pendulum in the upright position. To this end, a Linear Quadratic Regulator is designed that is optimal in the nominal case, i.e. not taking any sensing delays into consideration. Three sensors are used that measure the angle of the pendulum, the angular velocity of the pendulum, and the angular velocity of the base, respectively.

We then study the stability and performance of the closedloop system in presence of timing misalignments. First, we assess the stability when a constant delay is applied to each of the measurements. For a delay tuple  $(d_1, d_2, d_3)$ , we construct the corresponding Markov Jump Linear System. This system is mean square stable if and only if the spectral radius  $\rho$ associated to the system is smaller than 1. Figure 1 illustrates this for a range of delay values. One can immediately see that the system does not react in the same way to different delays. Even small values of  $d_2$  can destabilise the system, while the system is more robust to delays in the other sensor channels. Note that for this special case where the delays are constant, the analysis can as well be performed using standard methods. Our approach generalises the analysis to also cover non-constant delays.

Second, we consider the case where the sensor delay sequences follow normal distributions with mean  $d_i$  and standard deviation 1. We then evaluate a cost metric  $\tilde{J}(k)$ . The graphs of  $\tilde{J}(k)$  for some combinations of mean values are shown in Figure 2. It is worth noting that the asymptotic cost does not necessarily increase monotonically with the amount of delay.

## **IV. CONCLUSION**

We have provided a comprehensive analysis of the stability and performance of control systems subjected to sensor timing misalignments, a critical issue in practical applications like autonomous driving and aerospace. By modelling multichannel sensor delays as random variables, we formulate the problem using a Markov Jump Linear System framework. This enables the examination of mean square stability and expected performance. This work highlights the need for control designs that can effectively mitigate timing misalignment effects.

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