Optimal disturbance decoupling algorithms over networks

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Luca Claude Gino Lebon and Claudio Altafini *

Disturbance decoupling over networks. We consider a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of nodes $\mathcal{V} = \{v_1, \ldots, v_n\}$ to which we associate a linear control system with state update matrix equal to the (weighted) adjacency matrix A of the graph. We associate one state variable x_i of the vector $x = [x_1 \ldots x_n]^\top$ to each node v_i of the graph. We consider the following linear system

$$\begin{aligned} \dot{x} &= Ax + Bu + Dw \\ y &= Cx \\ z &= Tx. \end{aligned} \tag{1}$$

D and T are elementary matrices that identify, respectively, the sets of disturbances and targets acting on $\mathcal{G}$, denoted $\mathcal{D}$ and $\mathcal{T}$. Similarly, B and C are also elementary matrices that identify, respectively, the sets of control inputs and output nodes on $\mathcal{G}$, denoted $\mathcal{I}$ and $\mathcal{O}$. In our setting, $\mathcal{T}$ contains nodes of special importance in the network, and the task of this work is to find graphical conditions to isolate them from the effect of the disturbance w. Formally, we look for a control law $u(t)$ that brings the transfer function from the disturbance vector w to the vector of targets z to zero for all frequencies. This problem is an example of a disturbance decoupling problem (DDP) over networks. The conditions for solving the DDP available in geometric control vary depending on the control law taken into account [1]. Such conditions are based on controlled and conditional invariance, or on the existence of a so-called (C, A, B)-pair. The strategies under analysis are state feedback (DDPSF), output feedback (DDPOF), and dynamical feedback (DDPDF), whose specifics are summarized in Table 1. One contribution of this work is to reformulate these conditions in graphical terms, building on the setting proposed in [2]. Another is to identify the sets of input nodes $\mathcal{I}$ (and, when needed, of output nodes $\mathcal{O}$) that solve the DDP with minimum input (and output) cardinality.

Optimization problem reformulation. The simplest problem to solve is the DDPSF. Here, our task is to choose the set of control nodes $\mathcal{I}$ so that the existence of a state matrix F that decouples $\mathcal{T}$ from $\mathcal{D}$ is guaranteed. We show that a necessary and sufficient condition for decoupling is that each $\mathcal{D}$-to-$\mathcal{T}$ path has nonempty intersection with $\mathcal{I}$, i.e., at least a control node per path needs to be present. Assuming that the full state is

*L. C. G. Lebon and C. Altafini are with the Division of Automatic Control, Dept. of Electrical Engineering, Linköping University, SE-58183, Linköping, Sweden. email: luca.claude.gino.lebon@liu.se, claudio.altafini@liu.se

accessible at each time t can be quite unrealistic in practical applications. To circumnavigate such limitation, the second approach consists in solving the DDPOF, that is, computing $u = -Gy$, where the output vector y is from a second set of nodes $\mathcal{O}$ to be chosen. We show that the feasibility (that is, the existence of a matrix G that decouples $\mathcal{T}$ from $\mathcal{D}$) is guaranteed in this case by the existence of at least one sub-path of length 1 of the form $\{v_p, v_{p+1}\}$ with $v_p \in \mathcal{O}$ and $v_{p+1} \in \mathcal{I}$ for each $\mathcal{D}$-to-$\mathcal{T}$ path. A third approach consists of designing a dynamical system (a compensator), whose output is the control input that achieves the decoupling. Graphically, feasibility in this case is guaranteed if in each $\mathcal{D}$-to-$\mathcal{T}$ path (labeled locally so that it is sorted in ascending order from the node in $\mathcal{D}$ to the node in $\mathcal{T}$) there exist at least an output node and an input node and the output node closest to $\mathcal{D}$ appears before the input node closest to $\mathcal{T}$.
Each of the aforementioned disturbance-decoupling sub-problems admits a further graphical interpretation. Essentially, feasible solutions $\mathcal{I}$ (for DDPSF) or $\mathcal{I}$, $\mathcal{O}$ (for DDPOF and DDPDF) are cuts in $\mathcal{G}$ of the flows from $\mathcal{D}$ to $\mathcal{T}$. Within an optimization framework, the cost functions to be minimized are the edges to be cut to achieve feasibility or the cardinality of the input/output sets of nodes. In such conditions, we show that the optimal solutions can be computed in polynomial time by means of readaptations of the mincut/maxflow algorithms.

DDPSF	**DDPOF**	**DDPDF**
Static $u = -Fx$	Static $u = -Gy$	Dynamic $\dot{\hat{x}} = K\hat{x} + Ly$ $u = -M\hat{x} - Gy$
Optimization problem's cost functions		
$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{I}\rvert + \lvert\mathcal{O}\rvert$	$\lvert\mathcal{I}\rvert + \lvert\mathcal{O}\rvert$

Table 1: Key features of the disturbance decoupling sub-problems.

References

[1] Giuseppe Basile and Giovanni Marro. Controlled and conditioned invariant subspaces in linear system theory. *Journal of Optimization Theory and Applications*, 3:306–315, 1969.

[2] G Conte, AM Perdon, E Zattoni, and Claude H Moog. Invariance, controlled invariance and conditioned invariance in structured systems and applications to disturbance decoupling. In *IOP Conference Series: Materials Science and Engineering*, volume 707, page 012010. IOP Publishing, 2019.