Controllability Analysis of Multi-Modal Acoustic Particle Manipulation in One-Dimensional Standing Waves

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***Abstract*— Acoustic manipulation in microfluidic devices enables contactless handling of biological cells for Lab-on-Chip applications. This extended abstract analyzes the controllability of multi-particle systems in one-dimensional acoustic standing waves using multi-modal actuation. By modeling the system as a nonlinear control system, we investigate global and local controllability, quantifying the impact of mode numbers. Our findings demonstrate that sufficient modes ensure dense reachability sets globally, while mode mixing with 10 modes achieves local controllability in 80% of the state space for a two-particle system. These results provide theoretical insights for designing efficient acoustic manipulation algorithms in biomedical applications.**

***Index Terms*— Biotechnology, Acoustic manipulation, Controllability analysis**

I. Introduction

Lab-on-Chip (LOC) technologies are revolutionizing biomedical applications by enabling miniaturized, automated handling of biological cells. Acoustic manipulation, a contactless technique, is particularly promising for manipulating microscale particles and cells within microfluidic devices [1], [2]. Traditional acoustic manipulation relies on a single resonance mode, but recent advances in multi-modal actuation allow precise control of individual particle trajectories [3], [4], [5], potentially enabling single-cell manipulation at reduced costs compared to optical traps.

This work addresses the controllability of multi-particle systems in a one-dimensional (1D) acoustic standing wave, modeled as a nonlinear control system. We analyze global controllability among stable equilibria and local controllability via mode mixing, focusing on a two-particle system for clarity. Our contributions include constructing a controllability graph to demonstrate dense reachability with sufficient modes and quantifying local controllability regions through simulations. Fig. 1 illustrates the manipulation process, depicting particle configuration changes driven by a 1D standing wave.

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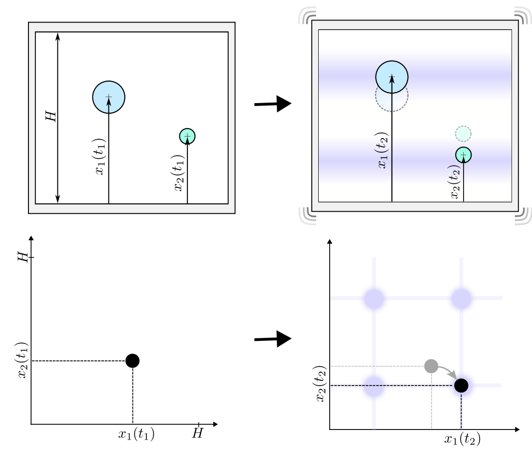

Fig. 1. Schematic of acoustic manipulation. **(Top)** Particle configuration changes due to a 1D standing wave from t_1 to t_2. **(Bottom)** State-space representation of the process.

II. System Modeling

We model the motion of n particles in a 1D acoustic standing wave within a channel of height H. The acoustic radiation force on a particle i at position $x_i(t) \in [0, H]$ is derived from the Gorkov potential [6], [7]:

$$U(x_i(t), u) = \frac{3}{2} V_i E_{\text{ac},u} \Phi_i \cos\left(2\pi \frac{u}{H} x_i(t)\right) + \text{const.},$$

where V_i, Φ_i, and $E_{\text{ac},u}$ are the particle volume, contrast factor, and mode energy, respectively, and $u \in \mathbb{N}$ is the mode number. Balancing acoustic and Stokes' drag forces, and neglecting inertia due to dominant viscous effects [8], the dynamics simplify to:

$$\dot{x}_i(t) = c_{i,u} u \sin\left(2\pi \frac{u}{H} x_i(t)\right), \quad c_{i,u} = \frac{\pi a_i^2 \Phi_i E_{\text{ac},u}}{2H\eta},$$

where a_i is the particle radius and η is the fluid viscosity. Scaling positions to $[0, 1]$ by x_i/H, the system for n particles is:

$$\dot{x}_i = A_i u \sin(2\pi u x_i), \quad x = [x_1, \ldots, x_n]^\top \in [0, 1]^n,$$

for all $u \in \{1, \ldots, N\}$, where A_i are constants (assumed mode-independent for controllability analysis, as justified by Lemma 1 in the original paper). The system is nonlinear and multi-stable, with stable and unstable equilibria determined by the mode u.

III. Controllability Analysis

A. Global Controllability

Global controllability is analyzed over assignable stable equilibria, defined as $E_k = \{(\frac{2i_1-1}{2k}, \dots, \frac{2i_n-1}{2k}) : i_j \in \{1, \dots, k\}\}$ for mode k, with $E^N = \cup_{k=1}^{N} E_k$. We construct a controllability graph G_N with vertices E^N and edges between equilibria reachable by applying a single mode. The graph's strongly connected components (SCCs) indicate reachable equilibria sets.

Fig. 2 shows SCCs for a two-particle system with $N = 8, 12$ modes, excluding diagonal effects (where $x_1 = x_2$). As N increases, the number of disconnected components decreases, and at $N = 9$, all equilibria in $\{0 < x_2 < x_1 < 1/2\}$ are reachable. Including diagonal effects (e.g., transitions like $E_3(1,1) \to E_3(1,2)$) enhances connectivity, making the graph strongly connected for $N = 9$ in $]0, 1/2[^2$ (Fig. 3). Theorem 1 (original paper) proves that with $A_1 \neq A_2$, reachability sets become dense as $N \to \infty$.

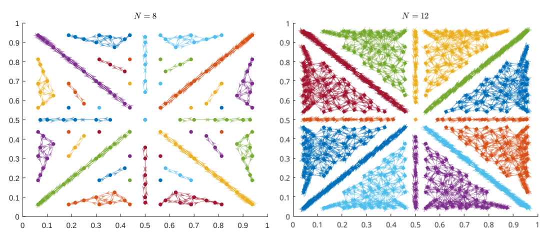

Fig. 2. Strongly connected components of the controllability graph for $N = 8, 12$ without diagonal effects.

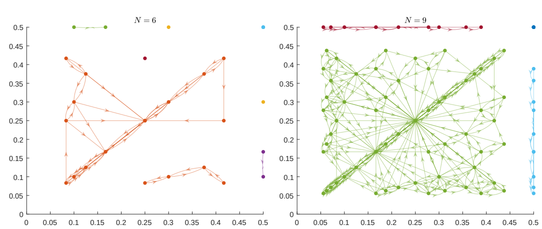

Fig. 3. Controllability graph with diagonal effects for $N = 6, 9$.

B. Local Controllability

Local controllability is studied via mode mixing, relaxing the system to a differential inclusion $\dot{x} \in \mathrm{co}(F(x))$, where $F(x) = \{f_u(x) = [A_1 u \sin(2\pi u x_1), \dots, A_n u \sin(2\pi u x_n)]^\top : u \in \{1, \dots, N\}\}$. A state x is locally controllable if $x \in \mathrm{int}\,\mathrm{co}(F(x))$, allowing movement in any direction. Using the Filippov-Ważewski theorem [9], the relaxed system's reachability approximates the original system's.

For a two-particle system ($a_1 = 1$ µm, $a_2 = 2$ µm, $H = 800$ µm), we discretize the state space into a 159x159 grid (5 µm spacing) and test local controllability. Fig. 4 shows that with $N = 5$, 58.4% of states are locally controllable; $N = 10$ achieves 80%. States on symmetry lines ($x_1 = x_2$ or $x_i = 1/2$) are uncontrollable due to colinear $f_u(x)$. Fig. **??** illustrates that for $p = 2$ to 10 particles, $N \geq p + 1$ is required for controllability, with diminishing returns as N increases for larger p.

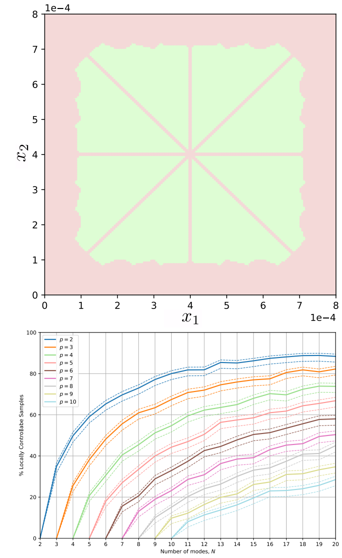

Fig. 4. Approximation of locally controllable states ($N = 5$, 58.4% controllable) in a 159x159 grid. Percentage of locally controllable states vs. N for $p = 2$ to 10 particles.

IV. Conclusion

This analysis establishes theoretical foundations for multi-modal acoustic manipulation in 1D standing waves. Global controllability ensures dense reachability with sufficient modes, while local controllability via mode mixing achieves 80% state space coverage with 10 modes in a two-particle system. These insights guide the design of efficient control algorithms for LOC applications. Future work will extend to multi-particle systems and experimental validation.

References

[1] A. Urbansky, F. Olm, S. Scheding, T. Laurell, and A. Lenshof, "Label-free separation of leukocyte subpopulations using high throughput multiplex acoustophoresis," *Lab on a Chip*, vol. 19, no. 8, pp. 1406–1416, 2019.

[2] C. Magnusson, P. Augustsson, E. Undvall Anand, A. Lenshof, A. Josefsson, K. Welén, A. Bjartell, Y. Ceder, H. Lilja, and T. Laurell, "Acoustic enrichment of heterogeneous circulating tumor cells and clusters from metastatic prostate cancer patients," *Analytical Chemistry*, vol. 96, no. 18, pp. 6914–6921, 2024.

[3] Q. Zhou, V. Sariola, K. Latifi, and V. Liimatainen, "Controlling the motion of multiple objects on a chladni plate," *Nature communications*, vol. 7, no. 1, p. 12764, 2016.

[4] Z. Shaglwf, B. Hammarström, D. Shona Laila, M. Hill, and P. Glynne-Jones, "Acoustofluidic particle steering," *The Journal of the Acoustical Society of America*, vol. 145, no. 2, pp. 945–955, 2019.

[5] K. Yiannacou and V. Sariola, "Acoustic manipulation of particles in microfluidic chips with an adaptive controller that models acoustic fields," *Advanced Intelligent Systems*, vol. 5, no. 9, p. 2300058, 2023.

[6] L. P. Gorkov, "On the forces acting on a small particle in an acoustical field in an ideal fluid," in *Sov. Phys.-Doklady*, vol. 6, 1962, pp. 773–775.

[7] H. Bruus, "Acoustofluidics 7: The acoustic radiation force on small particles," *Lab on a Chip*, vol. 12, no. 6, pp. 1014–1021, 2012.

[8] T. Baasch, I. Leibacher, and J. Dual, "Multibody dynamics in acoustophoresis," *The Journal of the Acoustical Society of America*, vol. 141, no. 3, pp. 1664–1674, 2017.

[9] J.-P. Aubin and A. Cellina, *Differential Inclusions: Set-Valued Maps and Viability Theory*, ser. Grundlehren Der Mathematischen Wissenschaften. Berlin New York: Springer, 1984, no. 264.