Automatic Portfolio Management framework using a Linear Quadratic Gaussian control law

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1. Introduction

Managing a portfolio of financial assets is a problem that many investors around the world must solve. This management is often affected by numerous cognitive, behavioral and emotional biases that can cause the initially defined final objective to be not so well addressed. By introducing an automated decision system for the purchase and sale of these financial assets, the author propose a new portfolio management framework that, while taking into account the latest performance observed on the financial markets, optimizes the investment and divestment decisions according to the known past performance, the estimated future trading costs, the average level of volatility of the portfolio over the entire remaining investment period and, finally, the expected final performance.
In this way, we propose to use a well-known control law to perform a dynamic optimization over a time horizon of the performance of a portfolio and can be considered as an extension of the "static" framework proposed by Harry Markovitz in his seminal paper (Markovitz 1952).
This communication will be structured in four parts and will be more focused on the “automatic control” point of view than the financial one. First, we will present the mathematical model used to represent the problem of optimizing the performance of a portfolio of financial assets. Then, we will describe the control law used and we will present why a Linear Quadratic Gaussian control seems to be more efficient than a Model Predictive Control law usually used in the literature (Boyd et al 2014). The notional and observed performances of the system will be presented from two portfolios of financial assets taking into account real stocks markets data between January 1997 and March 2025. We will conclude the presentation with the extensions that can be imagined and, in particular, the implementation of an adaptive control law.

2. Mathematical model for managing a portfolio of financial assets

In this section, we present the state model used to implement a Linear Quadratic Gaussian controller:

1. State vector x_t, input vector u_t and output PTF_t
2. Continuous state model without or with arbitrage between the assets:

$$(x_t^-) = R^{t\,|\,t-1} \text{x} \, (x_{t-1}^+)$$

$$(x_t^+) = R^{t\,|\,t-1} \text{x} \, (x_{t-1}^+) \quad + B \quad \text{x} \, (u_{t-1})$$

3. LQG control law $u_t = \mathrm{L}_t * x_t$,
4. Criterion $\mathrm{J}(\mathrm{nH}, \widetilde{x_{nH}})$ to be optimized for the calculation of arbitrage between assets :
    1. A quadratic sum - with the weighting matrix $Q_1(t)$ (n , n) - between t = 0 and t = nH-1 of states (x_t),
    2. A quadratic sum - with the weighting matrix $Q_2(t)$ (n-1 , n-1) - between t = 0 and t = nH-1 of the arbitrages (u_t),
    3. The deviation between the state x_{nH} and the target state $\widetilde{x_{nH}}$ with the weighting matrix Q_0 (n,n).

$$\mathrm{J}(\mathrm{nH}, \widetilde{x_{nH}}) = \sum_{t=0}^{nH-1} x_t^T \mathrm{Q}_1(\mathrm{t})\, x_t + \sum_{t=0}^{nH-1} u_t^T \mathrm{Q}_2(\mathrm{t})\, u_t + (x_{nH} - \widetilde{x_{nH}})^T \mathrm{Q}_0 (x_{nH} - \widetilde{x_{nH}})$$

3. Justification for the choice of a LQG controller

We deal with a multivariable system in a stochastic environment. Among the classical control laws, we find two main families of the most efficient multivariable controllers:

1. Pole placement controllers, such as Linear Quadratic Gaussian (LQG) controllers.
2. “Model Predictive Control” controllers using optimization and dynamic programming tools to manage constraints and non-linearities.

We'll show how the LQG model is better suited to the problem of portfolio management. Among the advantages put forward, we will outline :

1. The simplicity of the LQG model and its very good adaptation to asset managements,
2. The robustness of the LQG model and the underlying assumptions;
3. How to use an observer to take into account some non-linearities,
4. How to adapt the calculated trades to integrate additional functions

We shall conclude this part by discussing some tunings of the LQG controller concerning 3 points:

1. the choice of the weighting matrices used in the criterion J: Q_0, Q_1, Q_2,
2. the use of data conforming to the state model (called "notional") and real data from the financial markets,
3. the use of a sub-sampling for the realization of arbitrages,

4. Achieved performance

We’ll present the performance obtained from a historical perspective. We begin by explaining how we evaluate performance. Then we'll present the overall results obtained with two typical portfolio management portfolios: on the one hand, a portfolio of 40 of some performing stocks in the EURO STOXX 600, and on the other, a portfolio of financial assets mixing Exchanged Traded Funds replicating worldwide equity indices, bonds, real estate and commodities (gold).
The developed system offers many alternatives that need to be tested and confronted with real situations. Two alternatives have been tested and are presented in this paragraph:

1. the impact of a cap on the maximum amount of cash calls,
2. the impact of a "full load" go-live, i.e. an investment equal, from the first week, to the maximum notional amount foreseen rather than a progressive ramp-up from 0.

5. Conclusion and potential follow-up

In view of the performance obtained, we believe that this type of controller is more efficient, more intuitive to use by non-specialists in automatic control and, above all, much simpler to implement than the model-based approaches (Model Predictive Control) that have been presented in the scientific literature recently (Boyd et al 2014), (Syaifudin W., Putri E, 2019).
The potential applications of this work are many and varied. We have identified three, which we will present at the end of this presentation.

Bibliography

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