Multipolar dynamics of social segregation:

Data validation on Swedish vaccination statistics

Luka Baković, David Ohlin and Emma Tegling

***Abstract*—We perform a validation analysis on the multipolar model of opinion dynamics. A general methodology for using the model on datasets of two correlated variables is proposed and tested using data on the relationship between COVID-19 vaccination rates and political participation in Sweden. The model is shown to successfully capture the opinion segregation demonstrated by the data and spatial correlation of biases is demonstrated as necessary for the result.**

I. Introduction

A wide range of dynamical models have been proposed to capture social interactions and dynamics of opinions in large groups of people. The tutorials [1], [2] review dominant frameworks in the field and their respective area of application. These models all base the local dynamics of interpersonal interaction on sociological principles, while seeking to avoid undue complexity in an effort to mirror the simplicity of the physical laws of nature. As a consequence, rich analytic results have been derived and numerous extensions to different settings have been proposed in the literature; recent works include [3]–[5]. While some of the basic models have been empirically validated in limited settings (e.g. [6]) the same does not hold for the many offshoots and more complex variations.

Using the model first presented in [7], we aim to treat cases where the final distribution is spread out across the spectrum of different options, and the decision is individual. Relevant scenarios include, for example, general elections or media consumption. In this work, we apply the model to public data gathered by the governmental agency Statistics Sweden on two different issues: the rate of electoral participation and the coverage of COVID-19 vaccination [8].

II. Opinion-Dynamical model

Consider a number of agents, each represented by a state vector $\mathbf{x}^i \in \mathbb{R}^k_+$, at all times constrained to the unit simplex of dimension $k-1$, i.e. $||\mathbf{x}^i||_1 = 1$. This is to be interpreted as a distribution representing the preference of agent i over k mutually exclusive options. Additionally, each agent is assigned an individual bias vector $\mathbf{r}^i \in \mathbb{R}^k_+$. Each entry represents the predisposition of agent i toward the corresponding option. The agents are connected by an underlying communication graph, with $\mathcal{N}^i$ denoting the neighborhood of agent i. Let $R^i = \mathrm{diag}(\mathbf{r}^i)$. At each time step the dynamics propagate according to

$$\mathbf{x}^i(t+1) = \frac{\mathbf{x}^i(t) + R^i \sum_{j\in\mathcal{N}^i} \mathbf{x}^j(t)}{||\mathbf{x}^i(t) + R^i \sum_{j\in\mathcal{N}^i} \mathbf{x}^j(t)||_1}. \quad (1)$$

While this system is nonlinear and analysis of its convergence properties is far from trivial, numerical analysis show that the model generally converges to a single attractive fixed point depending on R and (in some cases) $\mathbf{x}(0)$, provided the underlying graph is strongly connected.

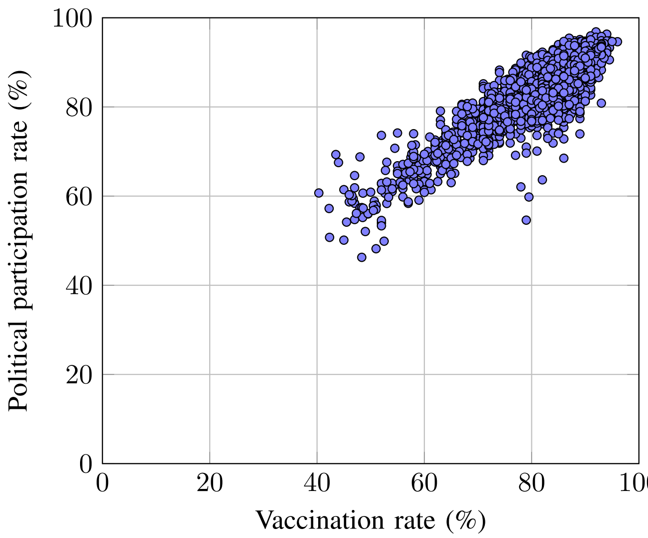

Fig. 1: The dataset [8] used for our analysis, dots represent regional statistical units

III. Dataset review

In 2022, Statistics Sweden published a report exploring the relationship between vaccination coverage and political participation [8]. Specifically, the percentage of people vaccinated against COVID-19 with at least two doses was compared to the percentage of people participating in the 2019 local elections. The data was collected on the level of demographic statistical regions (DeSO), the smallest geographical partitions of Sweden, which divide the country into 5984 units. Other divisions include 3363 regional statistical areas (RegSO) and 290 municipalities. The relationship between the variables was analyzed on all three levels and a positive correlation

The authors are with the Department of Automatic Control, Lund University, Lund, Sweden. They are members of the ELLIIT Strategic Research Area at Lund University. This work is partially funded by Wallenberg AI, Autonomous Systems and Software Program (WASP) funded by the Knut and Alice Wallenberg Foundation.
Email: {luka.bakovic,david.ohlin,emma.tegling}@control.lth.se.

was found. A scatter plot of the dataset can be seen in Figure 1. Furthermore, the dataset displayed an interesting spatial component. One finding was that regions with low vaccine coverage are also those with low political participation rates, whilst regions with high vaccination rates also display high voter turnout. Furthermore, regions with low vaccine coverage seem to be clustered, mostly on the outskirts of the three largest cities but also in certain areas of the countryside.

A. Numerical validation setup

This subsection describes the steps for applying the model to the SCB dataset. All simulations were performed using a Watts-Strogatz graph of just under 80000 nodes, randomly generated with the parameters $p = 0.2$ and $k = 8$. (The particular network size was chosen with regard to computation time, and could otherwise be much larger with a longer running time of the opinion dynamics algorithm as the trade off.) With our chosen neighborhood size and rewiring probability, the network exhibits an average path length of 5.86 making it closely resemble real social networks.

Graph neighborhoods were coupled to RegSO partitions in the following way. A list of all partitions and their populations was sorted by municipality. All partitions within a municipality were then given consecutive neighborhoods of the graph. As explained earlier, consecutive sequences of agents are well-connected with each other. Giving partitions within a municipality consecutive neighborhoods ensures that they also have a slightly higher connectivity internally, whilst different municipalities connect with each other almost uniformly. The sizes of each graph neighborhood were determined proportional to the fraction of the corresponding partition in relation to the whole population.

The vaccination rate was then used to distribute biases following the process described in the methodology section, with pro-vaccination agents $i \in \mathcal{V}_a$ being assigned a bias of $\mathbf{r}^i = [0.95\, 0.05]$ and anti-vaccination agents $j \in \mathcal{V}_b$ being assigned $\mathbf{r}^i = [0.05\, 0.95]$. The political participation rate for each geographical unit was then inferred from the average of the first opinion index in the corresponding unit of the graph.

IV. Results and discussion

A. Analysis of the output

A scatter plot of the simulated output can be seen in Figure 2. The model succeeds in capturing the overall shape of the original dataset seen in Figure 1. Namely, we can see a linear trend in the data along with a variance that increases in both directions away from the point where the vaccination rate is 65%. For lower values of the vaccination rate, the model predicts higher political participation rates than those exhibited by the dataset. It is important to note that this analysis relies on a correlation between the examined variables. Since this is assumed, the task of predicting the output variable in itself is of trivial importance. The point of the analysis, however, is to show that the dynamical model can indeed reproduce realistic results when parameters are appropriately chosen.

It is worth noting that established models of opinion dynamics do not usually exhibit such modes of convergence. Consensus models would give the result equal to a weighted average of the starting opinions. Using the bounded confidence model would result in several disjoint peaks in the distribution (consensus inside localized neighborhoods). Either of those would fail to capture the nature of the dataset.

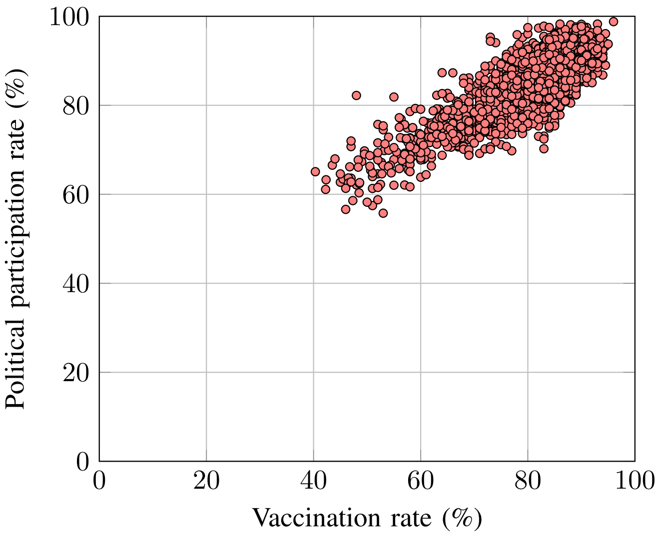

Fig. 2: Model output in the base case. The political participation rate is determined by the average of the first opinion index in each graph neighborhood corresponding to a regional statistical unit. Comparison to the measured data in Figure 1 shows a close resemblance of the overall shape.

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