Dissipative Barrier Feedback for Control of Safety-Critical Systems: Theory and Application

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I. INTRODUCTION

The control of large-scale safety-critical systems in a constrained environment is a critical challenge in intelligent transportation systems and Robotics, where robust and efficient solutions are essential. This abstract presents a novel safe control approach—Dissipative Barrier Feedback (DBF)— and its application to address the challenge of safe platoon formation control. Its constructive feedback control formulation enables a simple design by combining a nominal controller with the DBF, ensuring safety invariance as well as stability and convergence to the desired configuration. The abstract further exploits DBF to design a novel decentralized safe platoon formation controller with rigorous theoretical guarantees and practical validation through simulation studies. Additionally, an intuitive car-following example illustrates the nuances of the proposed approach in comparison to Control Barrier Functions (CBFs) and Hamilton-Jacobi (HJ) reachability, further demonstrating its effectiveness in enabling collision avoidance and justifying its usage for safe platoon formation.

II. DISSPATIVE BARRIER FEEDBACK FOR LARGE SCALE SAFETY CRITICAL SYSTEMS

Consider a multi-agent system under double intergrator dynamics

$$\begin{cases} \dot{p}_i = v_i \\ \dot{v}_i = u_i \end{cases} \tag{1}$$

where $p_i \in \mathbb{R}^2$ and $v_i \in \mathbb{R}^2$ are the position and velocity of agent i. The concept of Dissipative Barrier Feedback control is to design the control input as the sum of a nominal controller with a Dissipative Barrier Feedback

$$u_i = u_i^n + u_i^c, \tag{2}$$

where u_i^n is the nominal control input ensuring the asymptotic (or the exponential) convergence of the states (p_i, v_i) to the desired trajectory. u_i^c, is a *Dissipative Barrier Feedback* slowing down the relative velocity of agent i in the normal direction of the obstacles. Its effect vanishes when the relative velocity is orthogonal to the normal direction, ensuring that agent i's nominal motion is not altered. For a multi-agent system operating in a free space, to guarantee inter-agent collision avoidance, the *Dissipative Barrier Feedback* is designed as [1]

$$u_i^c = \sum_{j\in\mathcal{N}_i} k_o g_{ij}\phi_{ij}, \tag{3}$$

where $\mathbf{g}_{ij} := \frac{\mathbf{p}_j - \mathbf{p}_i}{\|\mathbf{p}_j - \mathbf{p}_i\|} \in \mathbb{S}^1$ is the unit direction vector from agent i to j, $\phi_{ij} := \frac{\dot{d}_{ij}}{d_{ij}}$ is the divergent flow, $d_{ij} := \|p_i - p_j\| - \epsilon$ is the safety distance between two agent i and j with $\epsilon > 0$ a safe margin, and k_o is a constant gain. The divergent flow ϕ_{ij} can be obtained directly from the optical flow using visual information, or estimated from the measure of d_{ij}.

To illustrate the safety invariance principle employed in this context, let's consider a 2-agent system under a directed graph (i.e., $\mathcal{N}_2 = i = \{j\}, \mathcal{N}_j = \varnothing$). For the sake of simplified notation, denote $d = d_{ij}$, $\dot{d} = \dot{d}_{ij} = g_{ij}^\top(\mathbf{v}_j - \mathbf{v}_i)$. One concludes that

$$\ddot{d} = -k_o\frac{\dot{d}}{d} - \alpha(t) \tag{4}$$

where

$$\alpha(t) = -\frac{\|\pi_{g_{ij}}(v_i - v_j)\|^2}{d+\epsilon} - g_{ij}^\top(u_i^n - u_j),$$

The safety invariance property provided by $\mathbf{u}_i^c$ is shown in the following lemma.

Lemma 1: Given the dynamics (4) with k_o a positive gain and $\alpha(t)$ a continuous and bounded function. Then for any initial condition satisfying $d(0) > 0$ and $\phi(0) = \frac{\dot{d}(0)}{d(0)}$ bounded, the following assertions hold:

1) d remains positive, $\forall t \geq 0$.
2) d converges to zero as $t \to \infty$ if and only if $\lim_{t\to\infty}\int_0^\top \alpha(\tau)d\tau \to +\infty$.
3) If d converges to zero, then $\dot{d}$ is bounded and converges to zero, and $\phi(t)$ remains bounded, $\forall t \geq 0$. Furthermore, if $\alpha(t)$ converges to a positive constant $\alpha^0 > \epsilon > 0$, then $\frac{\dot{d}}{d} \to -\frac{\alpha^0}{k_o}$ and hence $\ddot{d}$ converges to zero.

Proof of the lemma is given in [1]. This lemma shows the safety invariance property, i.e., as long as the initial distance $d(0)$ is positive, d will never cross zero for all times as long as the nominal controller u_i^n, the neighboring agent input u_j, and the relative velocity $v_i - v_j$ are continuous and bounded. Moreover, since only direction and relative distance measurements are required in the design of the disspative barrier feedback, the control methodology is suitable for large-scale safety-critical systems equipped with onboard local sensors operating in unknown environments. Despite computational efficiency, the proposed disspative barrier feedback offer

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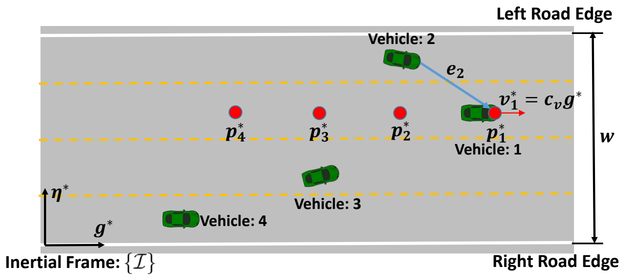


Fig. 1: Platoon formation scenario in a multi-lane highway segment at a certain time instance. The red dot indicates the desired position for each vehicle. The red arrow indicates the desired velocity for the platoon leader. The blue arrow between vehicle 2 and vehicle 1 indicates the relative position vector e_2.

promises to guarantee convergence to the target configuration without introducing local minima [1].

III. Application to Safe Platoon Formation Control in Constrainted Space

In this section, we will explore DBF to safe platoon formation and merging control problems for a group of Connected and Automated Vehicles in a multi-lane highway scenario, with environmental constraints such as limited free space with road edges.

The control objective is to design decentralized controllers for a group of vehicles under a leader-follower interaction topology to track the desired platoon formation while keeping a safe distance between neighboring vehicles and toward the road edges, as shown in Fig. 1

To drive the vehicle i to the desired lane while keeping a desired distance to the forward neighboring vehicle $i-1$, we decouple the control along longitude and lateral directions ρ and η, respectively, and design the nominal controller as

$$u_i^n = k_1 \rho \rho^\top (\tilde{e}_i + \nu_i) - k_2 \eta \eta^\top (\tilde{p}_i + \tilde{v}_i) + u_{i-1}, i \geq 2. \quad (5)$$

k_1 and k_2 are positive gains, $\tilde{p}_i := p_i - p_i^*$, $\tilde{v}_i := v_i - v_i^*$, and $\tilde{e}_i := e_i - e_i^* = \tilde{p}_{i-1} - \tilde{p}_i$ are absolute position error, absolute velocity error and relative position error of agent i, respectively.

To prevent collision between two neighbor vehicles i and $i-1$, the inter-vehicle safety distance

$$d_i = \|e_i\| - r \quad (6)$$

should be guaranteed all the time positive. Here, we explore inter-agent distance projected to the longitude direction of the road ρ: $d_i^\rho := \rho^\top e_i - \epsilon$. Since $\|e_i\| = \sqrt{(\rho^\top e_i)^2 + (\eta^\top e_i)^2}$, it is straightforward to verify that $d_i^\rho > 0$ implies $d_i > 0$.

Besides collision avoidance with the neighboring vehicle, all vehicles must follow the traffic rules so the road edges can not be exceeded. To prevent vehicles from crossing the two road edges, we define the safety distance to the road edges as $d_i^\eta := b_i - \epsilon$ with b_i the minimum distance between the vehicle i and the two road edges

$$b_i := \begin{cases} \eta^\top p_i, \ \eta^\top p_i \leq \frac{w}{2}, \\ w - \eta^\top p_i, \ \text{otherwise}, \end{cases} \quad (7)$$

where w is the width of the road.

The DBF u_i^c for the purpose of collision avoidance is designed as the sum of two terms

$$u_i^c = k_3 \rho \phi_i^l - k_4 \beta_i(t) \eta \phi_i^\eta \quad (8)$$

where $\phi_i^l =: \frac{\dot{d}_i^\rho}{d_i^\rho}$ with $\dot{d}_i^\rho = \rho^\top \nu_i$, serving for collision avoidance between neighboring agents; $\phi_i^\eta =: \frac{\dot{d}_i^\eta}{d_i^\eta}$ for collision avoidance against road edges where $\dot{d}_i^\eta = \beta_i(t) \eta^\top v_i$, and

$$\beta_i(t) := \begin{cases} 1, \ \eta^\top p_i \leq \frac{w}{2}, \\ -1, \ \text{otherwise}. \end{cases} \quad (9)$$

Theorem 1: Consider an n-agent ($n \geq 2$) system with the dynamics (1) along with the feedback control law (2), (5), and (8). For any safe and bounded initial conditions $(\tilde{p}_i(0), \tilde{v}_i(0))$ such that $d_i^\rho(0) > 0$ and $d_i^\eta(0) > 0$, $\phi_i^\rho(0)$ and $\phi_i^\eta(0)$ are bounded, the following assertions hold $\forall i \in \mathcal{V}/\{1\}$, $\forall t \geq 0$:

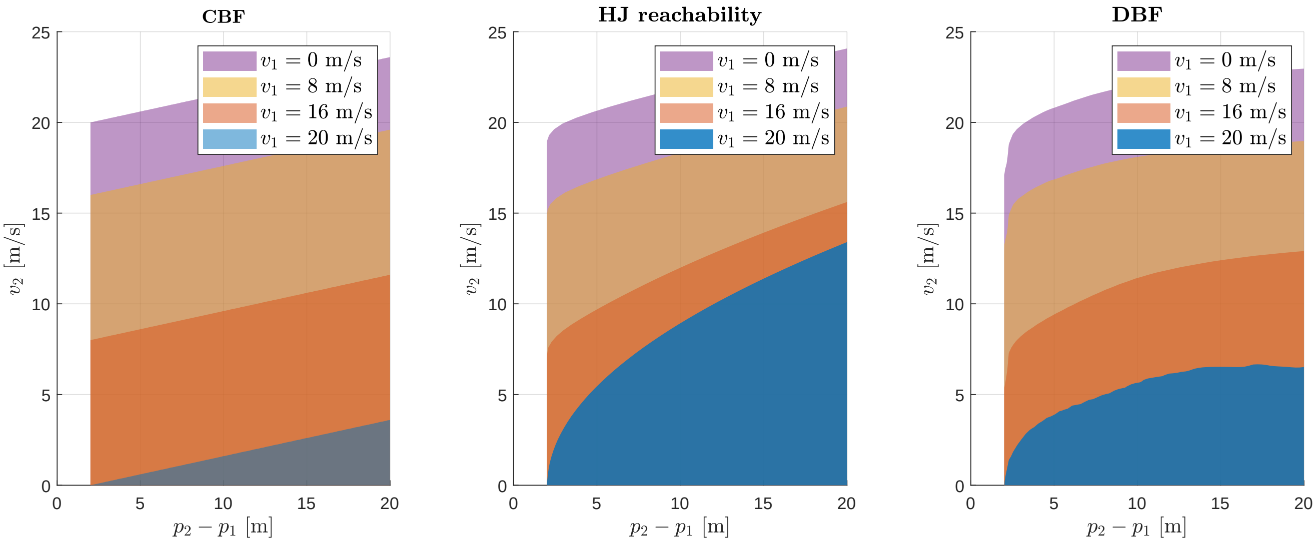
1) the n-agent system remains safe, i.e., $d_i(t)$ and $d_i^\eta(t)$ remains positive and $\phi_i^\rho(t)$, $\phi_i^\eta(t)$, and u_i are bounded,
2) the desired equilibrium point $(\tilde{p}_i, \tilde{v}_i) = (0, 0)$ is asymptotically stable.

The detailed proof of the theorem can be found in [2].

IV. Simulation

A comprehensive simulation results of the proposed method can be found in [2]. In this abstract, we will focus on the comparison with state-of-the-art optimization-based safety control methods. For clarity and conciseness, we focus on a simplified car-following scenario involving two vehicles in the longitudinal direction (i.e., one-dimensional space) in free space, rather than the full multi-agent platoon formation in constrained space.

We compare DBF with optimization-based safe controller, namely control barrier function and HJ reachability. Fig. 2 shows the approximated control-invariant set under same actuator constraints. The invariant set for DBF is bigger than for the CBF, as shown in Fig. 2. Note that safety invariance is ensured under all the DBF, optimization-based CBF, and HJ reachability designs, providing the same nominal controller. However, the optimization-based approaches faces challenges, particularly in large-scale systems operating within constrained environments. These challenges include potential feasibility issues and high computational costs associated with real-time optimization. Furthermore, due to its optimization-based nature, analyzing the stability and convergence toward the desired configuration becomes more complex and demanding. In contrast, the DBF-based approach offers a simple yet elegant solution that ensures safety while enabling formal analysis of stability and convergence. This makes it particularly well-suited for addressing the control of large-scale systems operating in constrained environments.



(a) CBF (b) HJ reachability (c) DBF

Fig. 2: The estimated control invariant set for the car following model in $(p_2 - p_1, v_2)$ space for different value of v_1, under same actuator constraints. Subfigure a) shows the set for CBF, subfigure b) shows the set for HJ reachability, and subfigure c) shows the set for DBF. Observe that the safe set for lower leader velocity v_1 is overlapped by that of higher v_1.

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