Using Heterogeneous Control Strategies to Achieve Length-Independent Behavior in Platoons

Ask Hällström[1]

I. INTRODUCTION

Modern control systems are often large and complex, consisting of multiple interconnected controllers and processes. A simple case of such systems is in platooning problems, where controllers and processes are arranged in a linear structure. This setup is commonly used to model vehicle platoons, but also appears in other applications such as skyscrapers with coupled oscillating floors [1], or in power grid and robotic swarm applications.

A problem often arising in large scale platoons is the emergence of resonance. Since the controllers often work with limited, local information the performance might be good on short scales such as keeping neighboring processes in sync with each other, while almost no control action is taken to mitigate long scale oscillations [2]. Another, closely related issue is that of instability. When the individual processes are unstable, extending the platoon indefinitely typically renders the entire system unstabilizable [3].

A common modeling assumption in the literature is that the controllers and processes are identical for all members of the platoon, i.e. that the platoon is homogeneous. In contrast, this work explores how a just small bit of heterogeneity can be exploited to change the system behavior. Specifically, we show how modifying just the last controller in an otherwise homogeneous platoon can enforce length-independent behavior. This builds upon the results in [4], extending them to a broader and more general framework.

II. MODELING

We model platoons as a chain of connected controller-process pairs, with n processes p_k and n controller c_k for $k = 1, ..., n$. In this work we consider symmetric bidirectional control, where both controllers and processes are SISO systems and each controller is fed the difference in output between their corresponding process and the neighboring ones. If the output of process k is denoted y_k the input to controller k becomes

$$(y_{k-1} - y_k) - (y_k - y_{k+1}) = y_{k-1} + y_{k+1} - 2y_k. \quad (1)$$

For the first platoon member a reference signal is used instead of one the outputs, and for the last member an offset g is used in the control law. When properly tuned this feedback loop leads to an averaging behavior in the platoon, where the processes in the case of vehicle platoons stabilize at a distance g from each other.

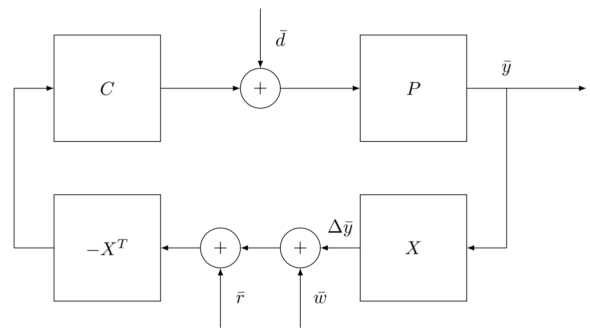

Fig. 1: Block diagram for the full platoon system.

By introducing matrices C, P and X the full system can be described by the block diagram in Fig. 1. The matrices C and P are simply diagonal matrices where the k:th diagonal entry corresponds to the transfer-function for process or controller number k respectively. The X matrix describes how the output signals from each process are feed back into the controllers according to (1), and is given by

$$X = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ -1 & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -1 & 1 \end{bmatrix}. \quad (2)$$

The signals $\bar{w}$ and $\bar{d}$ corresponds to measurement noise and external disturbances respectively, and the signal $\bar{r}$ handles both the reference value for the first platoon member, and the offset for the last. The outputs $\bar{y}$ and the relative outputs $\Delta\bar{y}$ are of interest from a control perspective and the transfer function from from example $\bar{r}$ to $\Delta\bar{y}$ is given by

$$\Delta\bar{Y} = -\left(I + XPCX^T\right)^{-1} XPCX^T\bar{R}, \quad (3)$$

where the factor $\left(I + XPCX^T\right)^{-1}$, is of importance. This factor shows up also in other transfer functions between signals in the control loop and can be seen as a sensitivity function for the system. A result related to this factor is therefore presented in the next section.

[1]Department of Automatic Control, Lund University, Sweden. The author is a members of the ELLIIT Strategic Research Area at Lund University. The author is grateful to the initiative COMPEL, with funding from the Swedish government

III. MAIN RESULT

Theorem 1: Let $n \in \mathbb{N}$ and let h and h_n be variables related according to $h = h_n(h_n+1)$. Furthermore let H and X be $n \times n$ matrices defined by

$$H = \begin{bmatrix} h & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & h & 0 \\ 0 & \dots & 0 & h_n \end{bmatrix}, \; X = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ -1 & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -1 & 1 \end{bmatrix}. \tag{4}$$

Then the inverse of $I + XHX^T$ exists if and only if $h_n \neq -1$, and has ij:th entry equal to

$$\frac{h_n^{i+j-2}}{(h_n+1)^{i+j-1}} + \sum_{k=2}^{\min(i,j)} \frac{h_n^{i+j-2k}}{(h_n+1)^{i+j-2(k-1)}}. \tag{5}$$

Remark 1: In the context of platoons, the theorem groups each process p_k and its corresponding controller c_k into a combined system $h_k = p_k c_k$. Homogeneity is then assumed for all $k < n$, so that $h_k = h$ for all such k smaller than n.

IV. DISCUSSION AND NUMERICAL RESULTS

Theorem 1 gives an explicit formula for the entries of $\left(I + XHX^T\right)^{-1}$ when the controller process pairs of the platoon follow a special relation, namely that $h_k = h_n(h_n+1)$ for all $k < n$. This essentially means that a special controller is used on the last platoon member, while the rest of the system is homogeneous. The benefit of this special controller becomes clear when studying the entries of the sensitivity function matrix, since these does no longer depend on n, as they would for an entirely homogeneous platoon. In practice this means that extending the platoon, can no longer affect earlier platoon members and that resonance is mitigated since it is typically length dependent. This change in system behavior can be seen in Figs. 2 and 3.

We can also conclude that the stability properties of the platoon can be improved using this special last member controller. The entries given in (5) are now all constructed from sums and products of the terms h_n and $(h_n+1)^{-1}$. If these are stable transfer functions non of the entries of the sensitivity function matrix will be unstable and then the platoon will be stable independently on n, in contrast to how increased platoon length often breaks stability for entirely homogeneous platoons.

V. CONCLUSION

To conclude, this work has shown how heterogeneity can be used to alter system behavior for platoons to achieve both length-independent system responses, and scalable stabilization properties. This was done by introducing a special control-process pair h_n as a last platoon member, related to the others by the formula $h = h_n(h_n+1)$. Numerical simulations confirms that this special control-process pair reduces resonance, and leads to length-independent behavior.

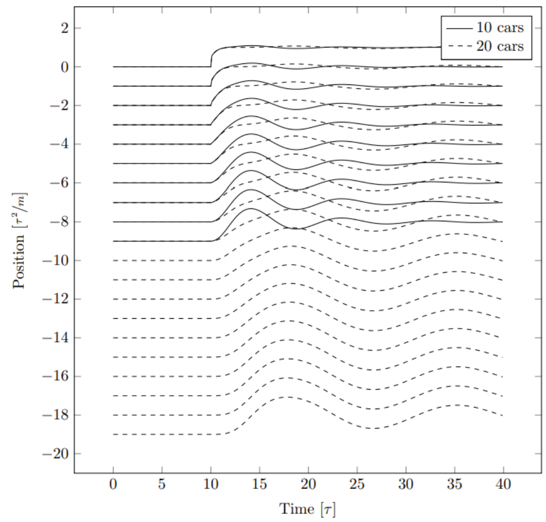

Fig. 2: The step response using homogeneous control for platoons of different length.

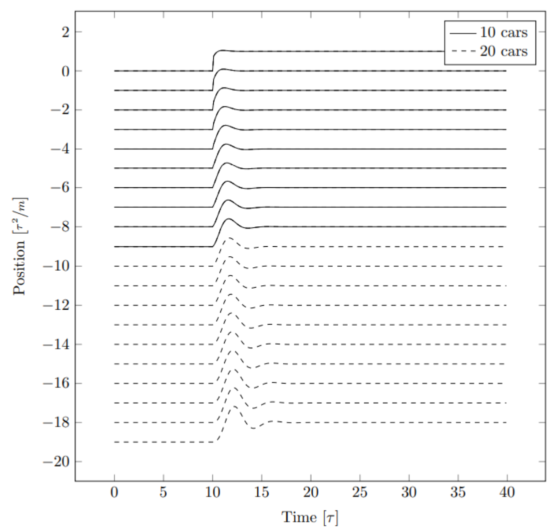

Fig. 3: The step response using heterogeneous control for platoons of different length.

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